4-2 Adding and Subtracting Polynomials

Objective: To add and subtract polynomials.

Vocabulary

Monomial An expression that is either a numeral, a variable, or the product of a numeral and one or more variables. For example: 13, m, $8c$, $2xy$, $5p^2q$.

Coefficient In the monomial $9x^2y^3$, 9 is the coefficient, or numerical coefficient.

Similar, or like, terms Two monomials that are exactly alike or the same except for their numerical coefficient. For example, $-3xy$ and $7xy$ are similar.

Constant monomial or constant A numeral such as 7.

Polynomial A sum of monomials. For example, $x^2 + 3x + y^2 + 2$.

Binomial A polynomial of only two terms. For example, $2x + 5$.

Trinomial A polynomial of only three terms. For example, $a^2 + 2ab + b^2$.

Simplified form, or simplest form, of a polynomial A polynomial which has no two of its terms similar.

CAUTION When a monomial does not have a written numerical coefficient, remember that its coefficient is 1. For example, $x^6y^2 = 1x^6y^2$.

Example 1 Simplify $-5x^3 + 2x^2 + x^2 + 7x^3 - 4$.

Solution
$$-5x^3 + 2x^2 + x^2 + 7x^3 - 4 = (-5x^3 + 7x^3) + (2x^2 + x^2) - 4$$
$$= (-5 + 7)x^3 + (2 + 1)x^2 - 4$$
$$= 2x^3 + 3x^2 - 4$$

Simplify.

1. $2x - y + 3x - 2y$

2. $7m - 5n - 2m + n$

3. $4x^2 - 3x - 2x^2 + 7x - 2$

4. $n^2 - 3n - 5n^2 + 7n + 6n^2$

5. $a^2 + 2ab - 5ab + 4a^2$

6. $x^2y - y^3 - 8x^2y + 5y^3$

7. $a^2b - 2ab^2 + 5a^3 - 3a^2b$

8. $-6xy^2 + 5x^2y - x^3 + xy^2 + 3x^3 - 2x^2y$

Example 2 Add $2x^2 + 5xy - 6y^2$ and $8x^2 + 6xy + y^2$.

Solution 1 First group similar terms and then combine them.
$$(2x^2 + 5xy - 6y^2) + (8x^2 + 6xy + y^2) = (2x^2 + 8x^2) + (5xy + 6xy) + (-6y^2 + y^2)$$
$$= 10x^2 + 11xy - 5y^2$$

Solution 2
$$\begin{aligned} 2x^2 + \ 5xy - 6y^2 \\ \underline{8x^2 + \ 6xy + \ y^2} \\ 10x^2 + 11xy - 5y^2 \end{aligned}$$
{ You can also align similar terms vertically and add. }

4–2 Adding and Subtracting Polynomials (continued)

Vocabulary

Degree of a variable in a monomial The number of times that the variable occurs as a factor in a monomial. For example, in $7x^3y$, the degree of x is 3, and the degree of y is 1.

Degree of a monomial The sum of the degrees of its variables. For example, the degree of $8x^2y^3$ is 5. The degree of any nonzero constant monomial, such as 10, is 0.

Degree of a polynomial The greatest of the degrees of its terms after it has been simplified. For example, the degree of $-5x^3 + 2x^2 + x^2 + 5x^3 - 4$ is 2, since it can be simplified to $3x^2 - 4$.

Add.

9. $3a - 1$
$4a + 3$

10. $4n + 2$
$-2n - 5$

11. $2x - 3y$
$-2x + 6y$

12. $5n - 2p$
$-3n + 5p$

13. $4x - 5y + 3$
$-2x + 7y + 7$

14. $2a - 3b - 6$
$3a - b + 8$

15. $6x^2 - 3x + 2$
$2x^2 + x - 5$

16. $5 - 2n - 6n^2$
$-3 + n - 2n^2$

17. $4c^2 - 3cd - 5d^2$
$-c^2 + 6cd - 2d^2$

18. $6a^2 - 2ab$
$-2a^2 + 5ab - b^2$

19. $3x - 2y - 5z + 1$
$2x + y - 3z$
$3y + z + 3$

20. $6a - 2b + 4$
$3a - 5c - 1$
$-a - b + 6c + 5$

Example 3   Subtract $-x^2 + 5xy + 6y^2 - 3$ from $3x^2 - 6xy - 2y^2 - 5$.

Solution 1   Add the opposite of $(-x^2 + 5xy + 6y^2 - 3)$ to $3x^2 - 6xy - 2y^2 - 5$.
$(3x^2 - 6xy - 2y^2 - 5) - (-x^2 + 5xy + 6y^2 - 3) =$
$(3x^2 - 6xy - 2y^2 - 5) + (x^2 - 5xy - 6y^2 + 3) = 4x^2 - 11xy - 8y^2 - 2$

Solution 2   You can also align similar terms vertically.

$3x^2 - 6xy - 2y^2 - 5$ Change to the $3x^2 - 6xy - 2y^2 - 5$
$-(-x^2 + 5xy + 6y^2 - 3)$ ⟶ opposite signs ⟶ $x^2 - 5xy - 6y^2 + 3$
                                                and add. $4x^2 - 11xy - 8y^2 - 2$

21–30. In Exercises 9–18, subtract the lower polynomial from the upper one.

Mixed Review Exercises

Simplify.

1. -2^3

2. $(-3)^2$

3. $2^2 + 3^2$

4. $(2 + 3)^2$

Solve.

5. $3(y + 2) - 2 = 2(4 - y)$

6. $10 = 2(n + 3)$

7. $4(x - 10) = 13 - 3(2x + 1)$

8. $-\frac{2}{5}(n + 3) = 10$

9. $c - 2 = |1 - 8|$

10. $\frac{3}{4}(2y - 6) = y - 7$